A Test for Collusion between a Bidder and an Auctioneer in Sealed-Bid Auctions

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Abstract

This paper derives a regression-based test to detect bidder-auctioneer cheating in sealed bid auctions. I apply this regression test to data from the New York City School Construction Authority auctions, an approximate one billion dollar per year auction market in which an auctioneer engaged in bidder-auctioneer cheating. Using the regression analysis to compare lots where bid rigging occurred with certainty to all other auctions allows one to conclude that bidder-auctioneer cheating significantly distorted the bid distribution. Comparing specific auctioneer lots before news of the cheating scandal became public with those after the scandal, I find significant differences in bidding, at the 10 percent level of significance, for two auctioneers. Therefore, bidder-auctioneer cheating may not have been limited to the one auctioneer charged with rigging bids.

Keywords: First-price auctions, collusion, bid-rigging, bidder-auctioneer cheating, order statistics, spacing

Introduction

Much has been written on the topic of bidder collusion, often called bidding rings or cartels, in auction markets. In a bidding ring, bidders agree before the auction to bid as a single entity. Such collusion reduces competition at the auction, and creates an artificial surplus that can then be divided among the ring members after the auction. Robinson (1985), Graham and Marshall (1987), and MacAfee and McMillan (1991) are examples of theoretical works on this subject. Porter and Zona (1993), Brannman (1996), and Baldwin, Marshall, and Richard (1997), offer statistical evidence for the presence of bidding rings in auction data. Until recently, the possibility of collusion between a bidder and an auction official has received little attention in

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the literature.[1] Yet, given the willingness of bidders to collude in auctions,[2] the potential for bidder-official collusion should be considered, because many large auctions (particularly government auctions) hire officials whose salaries are small relative to the prices of the items they auction off. Consequently, certain auction officials would enter a bid-rigging scheme in exchange for a sufficient kickback from a dishonest bidder. Also, a collusive equilibrium involving a bidding ring may not exist in a sealed-bid auction. In particular, ring members may not be informed of each other's bids, and therefore cannot punish those who break the ring (Robinson, 1985). Bidders that wish to organize a cartel to raise their profits might find such collusion impossible in a sealed bid environment. As an alternative, dishonest bidders may decide to bribe an auction official. Indeed, at least two incidents of bidder-auctioneer cheating have occurred in bidding markets for contracts in New York City.[3]

This paper contributes to the existing bid-rigging literature by analyzing a collusive mechanism between a bidder and an auctioneer. The mechanism is studied in both general terms and in the context of a specific case study—namely, the market for school repair and construction in New York City that is run by the New York City School Construction Authority (SCA hereafter). Section I defines the bidder-auctioneer cheating mechanism. Section II reviews the relevant literature, and Section III reviews the market for school construction and renovation in New York City. In Section IV, I present a model of bidder-auctioneer collusion, and then propose a statistical test to expose bidder-auctioneer cheating.

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[1] Andvig (1995) discusses bidder-auctioneer cheating as a form of favoritism that could exist in sealed-bid auctions for oil leases. Furthermore, Burguet and Perry (2000) and Menezes and Monteiro (2002) have working papers on bidder-auctioneer cheating in first-price sealed bid auctions. These papers are discussed below. Finally, Rothkopf and Harstad (1995) model auctioneer-only cheating in a Vickrey auction context.

[2] According to Porter and Zona (1993), more than 50% of the criminal cases filed between 1982 and 1988 by the Antitrust Division of the Department of Justice involved cheating in auction markets.

[3] Those two incidents are the bid-rigging scandal in the auctions for New York City School Construction Authority Contracts, which I examine in this paper, and the auctions for contracts to repair, demolish, or reconstruct low income housing in New York City that was collected for non-payment of taxes (Farber, 1988).

I. Bidder-Auctioneer Cheating and the "Magic Number"

Because the auctions for school construction contracts in New York are procurement auctions, this paper models bidder-auctioneer cheating within the context of a procurement auction. With trivial modifications, however, the model could be modified to reflect a value auction setting.[4] A procurement auction is an auction for a contract to perform a service for the auctioning body. The winning bid is therefore the lowest bid received rather than the highest bid received, as is the case in auctions for art, wine, or various collectibles.

The auction mechanism is a first-price, sealed bid auction. Contractors view their own private costs, and submit bids based on those costs. An auctioneer opens bids at a public ceremony, reading and recording bids. The lowest bid wins the contract, and the winning producer receives her bid as payment for completed work.

By bribing an auctioneer, a dishonest bidder could rig the outcome of a sealed-bid procurement auction. In particular, the dishonest bidder would persuade an auctioneer to falsify a bid on its behalf. The bidder would reveal its private costs to the auctioneer. The auctioneer would then agree to view all legal bids, and if the lowest legal bid were above the dishonest producer's cost, the auctioneer would submit a fake bid just below the lowest legal bid. Not only would the dishonest bidder increase its probability of winning that auction, it would also secure the contract at the highest possible price. This form of cheating will be referred to as "magic number" cheating. The magic number is the artificial bid the auctioneer would submit on behalf of the dishonest bidder.

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[4] In moving from a procurement auction to a value auction, the only real change in notation is that the probability of winning the auction is determined by the probability of bidding above other bidders rather than below them. Note, however, that this is strictly a mathematical argument. In general, different assumptions, such as common costs or affiliated values, are made on the underlying distributions when modeling a procurement market or a market of value. If different assumptions apply to a value auction, then obviously the results presented here will not apply directly to a value auctions.

For this type of cheating to be successful, it is necessary for the auctioneer to have complete control over the submitted bids long enough to doctor one bid, before any honest persons could view them. If the auction is private then the auctioneer would also own and police the market, and therefore the opportunity for this type of cheating should exist. If the auction is public, however, and the auctioneer is a government employee, then the opportunity for magic number cheating would depend upon how well the agency monitors its auctioneers.

II. Bid-Rigging Literature

The economics literature on the theory of bid-rigging and on the detection of bid-rigging in auction data is vast. Furthermore, several theory articles have recently been written on the collusion between a bidder and an auctioneer. Although I am unaware of any empirical papers on the topic of bidder-auctioneer cheating, the empirical bid-rigging literature provides insight to the proper manner to detect such collusion.

A. Theoretical

Burguet and Perry (2001) and Menezes and Monteiro (2002) have written a working paper on the subject of bidder-auctioneer collusion. Burguet and Perry consider a procurement auction market in which a dishonest seller competing against an honest seller has the option of bribing the buyer.[5] If a bribe occurs, the auctioneer will submit a rigged bid for the dishonest bidder, and the buyer will receive a percent of the dishonest bidder's profits in exchange.[6] The authors find that bribery can correct inefficiency in the auction if the dishonest bidder has a stochastic cost advantage over its opponent. Alternatively, added inefficiency results if the high-

[5] Because the market is one of procurement, the auctioneer is a government entity that is purchasing services from a list of potential suppliers—namely the bidders.

[6] The auctioneer submits the fake bid only if the dishonest bidder's cost is less than the honest bidder's bid.

cost bidder is dishonest. Finally, the authors find that the effect of briber on auction prices is ambiguous.

Menezes and Monteiro (2002) and have also studied bidder-auctioneer cheating. In their working paper, Menezes and Monteiro analyze bidder-auctioneer cheating in a value-auction setting with *n* risk neutral bidders with independent private values. The possibility of bribery occurs in every auction in that the auctioneer approaches the winning bidder and offering him a chance to lower his winning bid. Under both fixed and proportional bribery schemes they derive, when possible, the equilibrium, symmetric bid function. When this function exists, the auction is still efficient.

Other papers also study bribery schemes between a bidder and an auctioneer. Andvig (1995) considered the possibility of a bidder in North Sea oil auctions bribing an auction official in exchange for proprietary information. Arozamena and Weinschelbaum (2004) study the conditions under which a bidder will adjust its bid when it believes an opponent is bribing the auctioneer. Finally, Lengwiler and Wolfstetter (2000) analyze the effects of bidder-auctioneer cheating on auction bids and auction revenues.

B. Empirical

Although the theoretical literature on collusion in auctions exceeds in size the empirical literature on the topic, a growing body of empirical research has recently been written. Porter and Zona (1993), offer statistical tests for the detection of bidding rings that compare non-collusive auction lots to those where collusion is know to have existed. The authors have *a priori* knowledge of a bidding ring in the market for highway contracts in New York and know the identities of all cartel members. Chow tests reveal statistical differences in the estimated parameters between least-squares regressions of bid functions, and a multinomial logit model of bid rankings reveals statistical differences in low versus high ranking bids. The authors are then able to reject the null hypothesis that bid behavior is similar between the cartel and non-cartel samples. The authors conclude that the bidding ring did in fact distort the auction from a competitive market.

Bajari and Summers (2002) survey the empirical research on collusion in procurement auctions, and Fienstein, Block and Nold (1985) study the anti-competitive effects of bidding ring in the market for road construction contracts in North Carolina. Baldwin, Marshall, and Richard (1997) use a maximum likelihood model to detect evidence of collusion at U.S. Forest Service timber auctions while controlling for supply-side effects. Furthermore, Pesendorfer (2000) analyzes collusion in the procurement markets for school milk contracts in Texas and Florida.

III. Industry for School Construction and Renovation in New York City

The New York City School Construction Authority was created to improve upon the corruption and ineptitude that existed within its predecessor, the New York City Division of Buildings. By 1988, the Division of Buildings was the third largest maintenance office in the United States, after the maintenance offices of the Pentagon and the U.S. Postal Service.[7] In 1986 and 1987, nearly 30 inspectors and officials within the Division of Buildings were charged with bribery related crimes. Furthermore, the Division of Buildings required as much as 19 months to respond to what were often small maintenance requests, such as fixing a broken window (Dillon, August 15, 1993). Consequently, the School Construction Authority was

[7] The Division of Buildings was responsible for construction and maintenance in over 1,000 school buildings (Dillon, August 15, 1993).

created to handle the contracting and supervision of new school construction and major renovation and rehabilitation projects.

The major initiatives of the School Construction Authority included (1) assigning only one project manager to a job site, (2) paying contractors within 30 days after successful completion of a phase of a project, and (3) creating an inspector general's office with the power to investigate corruption. The first initiative was established to reduce arguments among project managers that had been assigned to the same job. It was thought that by having only one manager on site, change orders that were required would be granted more quickly, and the rate of project completion would be improved. The School Construction Authority decided to set the second guideline because in the past it had taken as long as a year for firms to be paid for completed work. As a result, many qualified firms refused to bid for projects. Finally, the logic behind having an inspector general's office was simple: police and reduce corruption in the industry (Raab, 1989).[8]

A. The School Construction Authority Auction Process

The School Construction Authority used first-price sealed bid auctions to allocate contracts. Auctioned contracts range in scale from the installation of a handicap sink (an estimated $6,700 job) to the construction of a new school (an estimated $85,000,000 job). Job sites are spread over all five New York City boroughs.[9] Most jobs are located in a single building, although some jobs require service at multiple buildings throughout a single borough, a

[8] One should note that oversight of the process is particularly important in light of the first initiative. In particular, multiple project managers make it more difficult to successfully engage in a change order scam—that is, changing the price of a component of a contract because of unforeseen complications in construction that are not the contractor's fault.

[9] Bronx, Brooklyn, Queens, Manhattan, and Staten Island.

combination of boroughs, or the entire city.[10] The main players in the auction scheme are the bidders, the contract specialists, and the project officers.

The bidders are construction firms. To bid on a project, a firm must first pre-qualify, which involves filling out a 22-page form detailing information about individuals in the firm, previous work completed for the city, past work experience, and a history of the ownership of the firm.[11] Bids submitted by firms that are not yet pre-qualified are immediately rejected. If the prequalification form is filled out incorrectly, all bids on all projects from that firm are rejected until the prequalification form is properly completed. Firms will be denied prequalification status if key members within the firm have recent criminal convictions, are under criminal investigation, or are suspected of "integrity problems." The inspector general's office at the School Construction Authority performs background checks on bidding firms and their employees.

The first official involved in the auction process is the contract specialist, an employee of the School Construction Authority. Contract specialists serve as auctioneers, opening sealed bids, recording those bids, ensuring that forms are completed properly, and ultimately assigning the job to a contractor. Contract specialists are not necessarily assigned to auctions based on job type (as is the case with project officers, as detailed below). Specialists are generally assigned between 10 and 40 contracts per fiscal year. Their main duty is to assign contracts quickly, and to the lowest-cost firm that is best suited to perform the work. Thus, if the lowest submitted bid is from a pre-qualified firm that correctly completed all its paperwork, including listing

[10] For example, many asbestos abatement jobs were contracted out for buildings across multiple boroughs.

[11] Subcontractors completing more than an estimated $10,000 worth of work on a project must be registered with SCA. If the value of their work is under $500,000, they may be only "sub-registered," and must fill out less paperwork than if they are to perform more than $500,000 worth of work, which requires prequalification status and the 22-page form that accompanies said status. Prequalification applications are available on the School Construction Authority's website at http://www.nycsca.org/html/forms.html#t_pre1296.

registered or pre-qualified subcontractors where necessary, then that firm is awarded the job. Otherwise, the specialist considers the integrity of the firm that submitted the next lowest bid. The process continues until a suitable contractor whose paperwork is in order secures the job. As a final note, there are occasions when contract specialists work together on a single auction, but these instances are very rare.

The second type of official employed at the School Construction Authority is the project officer. Project officers visit the site of a project and estimate its scale (often with the help of design plans from the SCA's engineering office) across all dimensions that would affect the cost of project completion. These measurements, and any design plans, are then made public to all firms wishing to submit a bid in the auction. The firms can either visit the SCA to view the plans, or they can order their own copies.[12] In addition, the project officer calculates a project estimate—that is, the estimated cost of completing the job. The estimate is recorded at the SCA but is *not* made public. Once a contractor has been assigned a job, the project officer routinely visits the job site to ensure that construction is completed in a satisfactory manner. Because of the nature of the project officer's job, officers are assigned to sites based on job type.

B. The Bid Rigging Scheme

During the summer of 1992, Elias Meris, the principal owner of the Meris Construction Corporation of Brooklyn, New York, was under investigation by the Internal Revenue Service. Seeking leniency from IRS agents, Meris offered to provide information on a bid-rigging scam involving School Construction Authority employees and other contractors. Working undercover for prosecutors, Meris taped conversations with senior project officer John Dransfield. As a consequence of those conversions, prosecutors charged both Dransfield, and his colleague, Mark

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[12] Costs range from about $50 to $250.

Parker, a contract specialist, who organized the scheme with bid-rigging. When confronted by authorities, Parker secretly pleaded guilty to charges of accepting bribes, and began working undercover to weed out contractors who were also on the take. Dransfield, however, pleaded not guilty to charges of accepting bribes until January 21, 1994 when he finally pleaded guilty to this charge (Raab, 1993; Fried, 1994). The Inspector General's office at the SCA estimated that Dransfield accepted $100,000 in bribes from the cheating scam and laundered the money through his Long Island construction firm (Fried, 1994). In addition to Parker and Dransfield, Samuel Manoharan, a project officer at the School Construction Authority, was charged with accepting bribes. In particular, Manoharan was charged with accepting between $3,000 and $4,000 in bribes for allowing a change order and for passing electrical work he should have failed (Olmstead, 1993).

Parker and Dranfield used magic-number cheating in their collusion scheme. Parker organized the scheme, and used Dransfield as an industry connection to recruit contractors into their group. Specifically, Dransfield would meet with the dishonest bidder prior to the bid submission. At that meeting, Dransfield suggested that the bidder submit a bid well below the projected price of the contract. At the public bid openings, Parker would save the dishonest contractor's bid for last, and knowing the current low bid, he would read aloud a false bid just below this price. Then, after the bid opening, Dransfield would use Wite-Out to doctor the bid form (Dwyer, April 21,1993). In this manner, the dishonest contractor is almost certain to win the contract, and will win the contract at the highest possible price (Olmstead, April 21, 1993). Furthermore, the School Construction Authority allowed bid withdrawals if a bidder believed that it erred in its cost calculations. Consequently, if Parker was being monitored at the bid opening and was unable to submit a false bid for that contractor, the contractor would not be required to complete the job for the low contract amount that he originally submitted.

Along with Parker and Dransfield, eleven individuals within seven contracting firms were implicated in the scam. These firms were Christ Gatzonis Electrical Contractor Inc., GTS Contracting Corp., Batex Contracting Corp., American Construction Management Corp., Wolff & Munier Inc., Simins Falotico Group, and CZK Construction Corp. (Olmstead, 1993; Raab, 1993). These seven firms won at least 43 SCA auctions with winning bids totaling over $23 million.[13]

IV. A Test for Magic Number Cheating

This section derives a regression specification that one may use to detect magic number cheating in sealed bid auctions. Because magic number cheating results in an artificial distortion of the distance between the first and second bids, a suitable test should compare the information available in the upper portion of the bid distribution to the distance between the two lowest bids. Below, I show that the third bid is the only bid from the upper portion of the bid distribution that need be included in the regression. This result is attractive, because it only restricts one's sample to auctions lots with at least three bidders.

A. Basic Model and Properties of the Ordered Bids

This section makes the assumption of pure private costs. In particular, assume that in a given auction lot, l, each bidder's cost is a random draw from a common distribution. Costs are private information, although each bidder knows the distribution from which costs are drawn.

[13] Some of these contracts were cancelled after criminal charges were made. In addition to the 43 winning contracts, these firms may have served as subcontractors on many further projects. However, the subcontractor data is unavailable.

Formally, each bidder, i, independently draws a cost, c_i, from a cost distribution $F(c)$, which is continuously distributed on the range $[\underline{c}, \overline{c}]$. N represents the number of bidders in the auction. Riley and Samuelson (1981) and Paarsch (1994), among others, show that the monotonic increasing bid function $B(.)$ results:

$$B(c_i) = c_i + \frac{\int_{c_i}^{\overline{c}} (1 - F(u))^{N-1} du}{(1 - F(c_i))^{N-1}} \tag{4.0}$$

Because $B(.)$ is a monotonic increasing function on $[\underline{c}, \overline{c}]$, the corresponding bids will also be randomly distributed, as shown in Guerre, Perrigne, and Vuong (2000). Therefore, for any given auction lot, log bids,[14] $B=\{B_1, B_2,\ldots, B_N\}$, are a random sample of size N from continuous density function $g(b)$ on $(b_{\min}, b_{\max})$, with associated distribution $G(b)$.

Because bids are analyzed in order, they are therefore order statistics. Define the *ordered* log bids as $B = \{S_1, S_2,\ldots, S_N\}$, where S_1 is the lowest-valued draw from $g(b)$, S_2 the second smallest, and so forth. The density and distribution functions of S_i, call them $f_i(s)$ and $F_i(s)$ respectively, are given in David (1981, 8-9):

$$\begin{aligned} &f_i(s) = \frac{N!}{(i-1)!(N-i)!} G(s)^{i-1} [1 - G(s)]^{N-i} g(s), \text{ for } b_{\min} < s < b_{\max} \\ &f_i(s) = 0, \text{ otherwise} \end{aligned} \tag{4.1}$$

[14] Because draws from a random sample can undergo any monotonic transformation and still maintain all their properties, if bids are drawn randomly then log bids are too. Thus, from this point on the analysis employs log bids only.

$$F_i(s) = \sum_{k=i}^{N} C_k^N G(s)^k [1-G(s)]^{N-K}, \text{ for } b_{\min} < s < b_{\max}$$

$$F_i(s) = 0, \text{ for } s < b_{\min} \quad (4.2)$$

$$F_i(s) = 1, \text{ for } b_{\max} < s$$

Because order statistics form a Markov process (David 1981, 20), the draws $S_1, S_2, \ldots, S_i$ can be seen as order statistics drawn from the density g(s) truncated on the right at the value y that exceeds S_i. For example, given $S_{i+1} = y$, S_i has a conditional density of

$$f_i(s \mid S_{i+1} = y) = \frac{i!}{(i-1)!} \frac{G(s)^{i-1} g(s)}{G(y)^i}, \text{ for } b_{\min} < s < y$$

$$f_i(s \mid S_{i+1} = y) = 0, \text{ otherwise} \quad (4.3)$$

Also, because of the Markovian nature of order statistics one can write the following:

$$f_i(s \mid S_{i+1} = s_1, S_{i+2} = s_2, \ldots, S_{i+N} = s_N) = f_i(s \mid S_{i+1} = s_1) \quad (4.4)$$

In words, a truncation at an order statistic higher than i+1 is meaningless if one has already truncated the density at i+1.

B. The Distribution of the First Spacing

Because magic number cheating distorts the distance between the first and second bids, we are therefore interested the difference between those two bids. The difference between two order statistics is called a spacing (David, 99). Define $D_{ij} = S_j - S_i$ as the difference between the *jth* and *ith* order statistics. The density of D_{ij} is given by equation 4.5 below.

$$f_{ij}(D_{ij}) = C_{ij} \int_{\infty}^{\infty} G(s)^i g(s)[G(s+D_{ij}) - G(s)]^{j-i-1} g(D_{ij}+s)[1-G(s+D_{ij})]^{N-j} ds \quad (4.5)$$

In the equation 4.5, N is the sample size, and C_{ij} is a constant equal to $\frac{N!}{(i-1)!(j-i-1)!(N-j)!}$ (David, 11).[15] The derivation of equation 4.5 is available in Pyke (1965). We can now modify the above equation to fit our particular case.

When studying magic number cheating the researcher is interested in the particular case where $i = 1$ and $j = 2$. Furthermore, if the third ordered bid is known, the first and second ordered bids are draws from the distribution $g(b)$ truncated at the third ordered bid. Hence, one is interested in the density of the first spacing when the sample size is two. Applying simple algebra to equation 4.5, and using $D = D_{21}$ for notational convenience yields the following:

$$f_{21}(D) = 2\int_{-\infty}^{\infty} g(s)g(D+s)ds \text{, for D > 0, and 0 otherwise.} \tag{4.6}$$

However, the underlying density from which we are drawing the *unordered* bids is the density $g(.)$, truncated on the right at the value y, where y is the realization of S_3. Therefore, unordered bids are drawn from $g(.)/G(y)$. Inserting this density into the above equation, and noting that $G(y)$ is a constant with respect to s, one can write the density of D conditioned on $S_3 = y$, written $f_{21}(D \mid S_3 = y)$, as

$$f_{21}(D \mid S_3 = y) = \frac{2}{G(y)^2}\int_0^y g(s)g(D+s)ds \text{, if } 0 < D < \text{y, and 0 otherwise} \tag{4.7}$$

Conditioning on other log bids in addition to the 3rd log bid will not change equation 4.7, as the density is already truncated at the 3rd log bid. Hence, the 3rd log bid is the only log bid that the researcher must condition on to explain the difference of the 2nd and 1st log bids.[16]

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[15] This is equation 2.3.1 on page 11 of David, 1981.

[16] Additionally, we should note that this property allows for maximal use of our sample data. If we condition on the 4th log bid as well, we must naturally omit lots with only three submitted bids, and will therefore lose efficiency.

C. A Model of Magic Number Cheating

Consider auction lots in which magic number cheating has occurred. One must first assume that if magic number cheating occurs, the dishonest bidder would have won the auction regardless. If this assumption is not made, then the observed bid ordering is not the true bid ordering, and the model is incorrectly specified. However, one should note that the gains from magic number cheating are highest when the dishonest bidder has the lowest cost, and would therefore win the auction absent collusion. Therefore, this assumption has some merit.

I model magic number cheating as a further truncation of the distribution of the first spacing. Specifically, after viewing $S_3 = y$, the auctioneer chooses a function $\theta(y)$, where $0 < \theta(y) < y$, and truncates the density of the first spacing at $\theta(y)$ rather than y. Thus, the density of this first spacing, when magic number cheating is present, can be written as

$$f_{21}(D \mid S_3 = y, collusion) = f_{21}(D \mid S_3 = \theta(y)) \tag{4.7}$$

Equation 4.7 holds for values within the ranges $(0, \theta(y))$. Consequently, the auctioneer maintains the random nature of the spacing between the lowest log bids, but reduces the range under which this spacing is drawn.

Two characteristics of equation 4.7 provide a logical test for magic number cheating:

1. The expected value of the first spacing, conditioned on the third log bid, decreases when magic number cheating exists.
2. If the collusive parties are insensitive to changing $\theta(y)$ when y changes, the marginal effect of a change in the third bid on the expectation of the first spacing is, most likely, negative.

A first-order stochastic dominance argument allows one to prove the first characteristic, and an application of Leibniz's rule to equation 4.7 allows one to verify the second. The appendix contains both analyses.

D. Regression Specification

Although the relationship between the log of the third bid and the conditional expectation of the first spacing will vary with the underlying bid distribution, if bids are distributed uniformly with a lower bound equal to 0, then the relationship will be linear. Therefore, a bid distribution exists that would justify a linear regression of the third bid on the first spacing—an attractive result indeed. Furthermore, one can show (see the appendix) that the upper bound of the uniform distribution is superfluous in the regression, so long as the third bid is included. In particular, because the distribution is already truncated at the third bid, the upper bound of the distribution is meaningless. Therefore, the upper bound of the bid distribution can vary across auction lots without affecting the regression specification. However, for other distributions where one can solve for the conditional expectation, such as the exponential distribution, the relationship between the first spacing and the third bid is non-linear. The appendix contains these derivations.

Define Y as an Lx1 vector of log differences between the second and first observed bids, where L is the total number of auction lots with at least 3 submitted bids. Define X as an LxK matrix of explanatory variables, including the log of the third bid, moments of the number of bidders, and various lot characteristics. However, conditioning on the third bid, in theory, eliminates the dependence of the density of the first spacing on the number of bidders (see equations 4.6 and 4.7 above). One can then perform the following regression using least squares:

$$Y_{21} = X\beta + \varepsilon \,, \tag{4.8}$$

where ε is an Lx1 disturbance vector with zero mean.

According to the first characteristic in section IV C, the expected value of the first spacing decreases in instances of magic number cheating. Therefore, if one knows the sample of collusive auction lots, one can perform the following regression to determine if magic number cheating alters bidding:

$$Y_{21} = Z\delta + e \tag{4.8}$$

In equation 4.8, Z is an Lxk+1 covariate matrix that is identical X, but for the addition of a 1-0 variable that identifies the collusive auction lots. If the coefficient on this indicator variable is negative and statistically significant, then one can conclude that magic number cheating significantly distorted the bid distribution in the direction suggested by characteristic 1 above. Consequently, a detection of magic number cheating amounts to a test for structural change in the regression coefficient between samples in which the researcher either knows or believes that magic number cheating has occurred.

V. Empirical Analysis of School Construction Authority Auctions

A. Data Summary

The complete School Construction Authority data set (including all auctioned contracts) represents 1,789 auctions held from May 1990 to January 1997. In these 1,789 auctions, just over $3 billion in contracts were awarded. Of these, 969 auctions representing a total of $2.29 billion in winning bids were competitive—that is, more than one contractor submitted a bid. Because the regression specification requires the 3rd lowest bid as an explanatory variable, the

890 auctions, with a sum of $2.24 billion in winning bids, in which at least three firms bid are relevant. Omitting the seven lots where I know magic number cheating occurred,[17] the sample then falls to 883 lots with a total of $2.23 billion in winning bids.[18] Table 1 contains a list of variable names used in the regression analysis.

TABLE 1: VARIABLE DEFINITIONS

ldev21	Log(2nd low bid) – log(winning bid)
lbid3	log(3rd low bid)
n	number of bidders
nsq	number of bidders squared
lncost	log(project estimate)
CS#	1 if lot was auctioned by a particular CS.
April21	1 if lot was auctioned before April 21, 1993.

The variable *April21* is used to control for possible collusion in auctions where I am unsure cheating has occurred. Table 2 includes a summary of the variables, other than contract specialist indicator variables, listed in Table 1, and the value of the winning bid for the sample of 883 lots. Finally, Table 3 provides summary statistics on the number of auction lots that contract specialists presided over in each of the three relevant samples.

[17] Although Parker plead guilty to rigging bids in eight auctions, I can identify only seven of the eight in the dataset.

[18] Compared to the typical auction dataset, 900 observations is quite large. Guerre, Perrigne and Vuong (1999) use samples of 200 in their Monte Carlo study explaining that this is a typical sample size for auction data.

TABLE 2: SUMMARY STATISTICS FOR REGRESSION SAMPLE:
N≥3 COLLUSION IS UNCERTAIN

Variable	Mean	Std	Min	Max
bid	2458000	6425000	3500	6.67e+07
ldev21	0.105	0.102	0	0.699
lbid3	13.302	1.709	8.7	18.05
n	6.929	4.131	3.0	28.0
nsq	65.07	89.77	9.0	784.0
lncost	13.293	1.72	8.52	18.26
April21	0.617	0.486	0.0	1.0

TABLE 3: SUMMARY OF CONTRACT SPECIALIST INDICATOR VARIABLES

	Total	x = 1	x = 0
# officers	22	16	14
ave # lots	38.45	31.81	24.07
std # lots	37.13	27.12	20.92
min # lots	1	1	1
max #	119	92	63
officer missing	37	36	1

B. Analysis of Auctions in which Cheating Occurred

The auction lots in which Parker was known to have engaged in magic number cheating are of specific interest. In particular, because one knows these auctions are tainted, one can control for these lots in the regression specification to determine if collusion resulted in a statistically significant change of the bid distribution. Table 4 contains results from least squares regressions of the first log spacing on the third log bid, other auction characteristics, and an indicator variable that equals one if magic number cheating is known to have occurred.

TABLE 4: ANALYSIS OF MAGIC NUMBER AUCTION LOTS:
DEPENDENT VARIABLE IS THE FIRST LOG SPACING

	Full Sample			**Parker Only Lots**		
Variable	**Coef**	**|t-stat|**	**|t-White|**	**Coef**	**|t-stat|**	**|t-White|**
lbid3	0.040**	4.08	3.53	-0.048	1.24	1.04
n	-0.009**	3.29	3.63	-0.024**	2.97	2.62
nsq	0.0003**	2.36	2.96	0.0007**	2.40	2.49
lncost	-0.054**	5.48	4.64	0.032	0.87	0.70
MN	-0.085**	2.33	8.40	-0.11**	2.52	4.83
Const	0.33**	12.35	12.45	0.45**	4.54	4.83
Obs		890	890		99	99
R^2		0.13	0.13		0.18	0.18
Adj R^2		0.13			0.14	

The regression results in Table 4 indicate that the first log spacing—that is the difference between the first and second log bids—was lower, on average, for lots in which magic number cheating is known to have occurred. Furthermore, this difference, which is -0.085 relative to the full sample of auctions, is statistically significant at the 95 percent level of confidence. One finds similar results when comparing magic number lots to the sample of auctions in which John Parker served as the contract specialist. Here the difference is -0.11, and that difference is also significant at the 95 percent level of confidence. Consequently, magic number cheating had a statistically significant affect on the bid distribution, controlling the third log bid and lot characteristics.

C. Auctioneer Specific Regressions Before and After April 21, 1993

After establishing that magic number cheating had a statistically significant impact on the distribution of ordered bids, a natural extension would be to determine if such cheating was confined only to the Parker auctions. Nine contract specialists oversaw a significant number of auctions before April 21, 1993—the date that news of Parker-Dransfield magic number scheme became public.[19] Additionally, only four of these nine contract specialists worked at the school construction authority after April 21, 1993. This fact will further limit the scope of required analysis.[20] Table 5 counts the auctions, by contract specialist, before and after April 21, 1993.

TABLE 5: COUNT OF AUCTIONS WITH AT LEAST THREE BIDDERS BEFORE AND AFTER APRIL 21, 1993

Specialist	Before April 21, 1993	After April 21, 1993	Last Auction
CS8	54	0	Summer, 1992
CS12	37	0	Summer, 1992
CS14	60	59	Unknown
CS17	55	22	1995
CS22	41	36	Unknown
CS27	16	0	Winter, 1991
CS30	47	0	Summer, 1992
CS32	55	63	Unknown
CS37	42	0	Summer, 1992
Parker	92	0	Approx March 1993

To begin our statistical analysis of the above contract specialists, we first run a regression controlling only for average affects before and after April 21, 1993 for specialists 14, 17, 22, and 32. These results are presented in Table 6.

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[19] There were 15 specialists before April 21, 1993, but 5 of these specialists only worked on one to five lots, and therefore their actions cannot be analyzed with any meaningful results expected. Additionally, one of the 15 specialists is Mark Parker, who we already know to be guilty. Thus, 9 specialists oversaw a significant number of auctions both before and after April 23, 1993.

[20] In a conversation with an SCA employee during June 2000, it was stated that SCA took measures to ensure that magic number cheating would never again occur. Given the ease with which this form of cheating could be eliminated—a simple monitoring device or public recording of bids as they are opened would be sufficient—it is safe to assume that cheating could only have occurred before April 21, 1993.

TABLE 6: CONTROLLING FOR AVERAGE AFFECTS, DEP. VAR.: LDEV21

Variable	**Coefficient**	**|t-stat|**	**|t-White|**
lbid3	0.035**	3.44	2.96
n	-.010**	3.47	3.83
nsq	0.0003**	2.55	3.23
lnpest	-.049**	4.89	4.11
CS32	0.017	1.33	2.05
CS17	0.018	0.85	1.44
CS14	0.012	0.88	0.71
CS22	0.018	1.05	0.17
April21xCS32	-0.036**	1.99	1.23
April21xCS17	0.031	1.29	0.90
April21xCS14	0.014	0.80	0.79
April21xCS22	0.003	0.15	1.11
cons	0.34	12.49	12.67
Obs		883	
R^2		.14	.14
Adj R^2		.12	

In Table 6, the estimated coefficient on CS32 is negative and significant at the 5% level. This result indicates that the average value of *ldev21* was lower in CS32 lots before April 21, 1993 than in the rest of the sample. According to section IV C, magic number cheating reduces the expected value of the first spacing. Therefore, CS32 could have engaged in magic number cheating before the bid-rigging scandal. Next, note that interactions between *April21* and the other CS indicator variables are negative but insignificant statistically.[21] Thus, the regressions in Table 6 suggest only that CS32 engaged in magic number cheating. To expand upon this analysis, I now run regressions on auction lots particular to contract specialist 14, 17, 22, and 32 to detect any within auctioneer sample evidence of cheating. Tables 7 and 8 contain the regression output.

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[21] Note that coefficients on dummies for specialists 8, 12, 27, 30, 37, and Parker are never significant. Additionally, the coefficients on CS8 and CS27 are negative while others are positive. Controlling for these specialists and those with lots pre and post scandal will still yield evidence supporting MNC under CS32 pre-scandal auctions, as the coefficient on xCS32 is still negative and significant at the 5% level.

TABLE 7: CONTROLLING FOR THE AVERAGE EFFECT OF APRIL 21, 1993:
CS32 AND CS17 SAMPLES, LDEV21 IS THE DEPENDENT VARIABLE

	CS32 Sample			CS17 Sample		
Variable	**Coef**	**|t-stat|**	**|t-White|**	**Coef**	**|t-stat|**	**|t-White|**
lbid3	0.023	0.79	0.58	0.065*	1.69	1.59
n	-.007	0.76	0.98	-.001	0.11	0.16
nsq	0.0001	0.31	0.43	-.00008	0.17	0.26
lnpest	-.037	1.27	0.93	-.087**	2.28	2.21
April21	-.037*	1.81	1.93	-.037*	1.69	1.50
cons	0.35	4.08	5.08	0.43**	3.96	3.78
Obs		118			77	
R^2		.14	.14		.26	.26
Adj R^2		.10			.21	

TABLE 8: CONTROLLING FOR THE AVERAGE EFFECT OF APRIL 21, 1993:
CS22 AND CS14 SAMPLES, LDEV21 IS THE DEPENDENT VARIABLE

	CS22 Sample			CS14 Sample		
Variable	**Coef**	**|t-stat|**	**|t-White|**	**Coef**	**|t-stat|**	**|t-White|**
lbid3	0.037	0.86	0.81	0.047	1.25	0.85
n	-.029**	3.40	3.62	-.023*	1.93	1.83
nsq	0.001**	3.08	3.93	0.001*	1.76	1.83
lnpest	-.057	1.33	1.20	-.057	1.55	1.04
April21	-.002	0.09	0.09	-.014	0.63	0.57
cons	0.51**	6.01	6.83	0.34**	3.46	3.59
Obs		77			119	
R^2		.37	.37		.11	.11
Adj R^2		32			.07	

The regressions in Table 7 indicate, albeit at the 10 percent level of significance, that contract specialists 17 and 32 may have engaged in magic number cheating. In particular, the indicator variable *April21* is negative and significant at the 10 percent level in both these regressions. Therefore, after controlling for the third bid, the first spacing was, on average, smaller before April 21, 1993 for both these contract specialists, which, according to section IV

C, is characteristic of magic number cheating.[22] In Table 9, the estimated coefficient on the indicator variable *April21* is negative for both CS14 and CS22, but that coefficient is not statistically significant in either sample. Hence, there is no evidence to suggest magic number cheating in auction lots monitored by CS14 or CS22.

Conclusion

This paper developed a regression model specifically designed to aid in the detection of magic number cheating between a bidder and auctioneers. Because the third ordered bid truncates the distribution from which the first spacing is drawn, an empirical analysis of magic number cheating need not include bids other than third ordered bid. Furthermore, a model of magic number cheating indicates that this form of collusion reduces the expected value of the first spacing, which advocates the use of a test for structural change in the detection of magic number cheating.

The regression model is applied to data from New York City School Construction Authority auctions for years 1990 through January 1997, and the analysis suggests that two auctioneers (other than the auctioneer already convicted) may have been guilty of magic number cheating. The regression method does appear to effectively detect magic number cheating, but we have only 7 auction lots where this form of cheating is known to have occurred. Therefore, conclusive evidence that this test is effective in unearthing guilty auctioneers is not yet available.

[22] If the indicator variable enters the regression in Table 7 as an interaction between *April21* and *lbid3* (which would be suggested by the second characteristic of magic number cheating from section IV C, the estimated coefficient is negative and significant at the 11% level for the CS32 sample and at the 7% level in the CS17 sample. These results further indicated that magic number cheating may have taken place in CS32 and CS17 monitored auctions. In the CS14 and CS22 samples, however, the coefficients on the interaction between *April21* and *lbid3* are never significant at, or near, the 10% level of confidence. Consequently, these results are again consistent with those presented in tables 7 and 8.

The next step in assessing the regression model's sensitivity to MNC is probably a Monte Carlo study aimed at the specific form of cheating described in this paper.

Appendix

A. Proof that $E(D \mid S_3 = y) \geq E(D \mid S_3 = \theta(y))$

First, note that for $\theta(y) < y$ one can write $0 < G(\theta(y)) < G(y) < 1$, for values of *θ(y)* and *y* in the range [b_{min}, b_{max}]. One can now write $\int_0^\infty \frac{g(s)g(D+s)}{G(y)^2} ds \leq \int_0^\infty \frac{g(s)g(D+s)}{G(\theta(y))^2} ds$, which in turn means that $\int_0^x \int_0^\infty \frac{g(s)g(D+s)}{G(y)^2} dsdD \leq \int_0^x \int_0^\infty \frac{g(s)g(D+s)}{G(\theta(y))^2} dsdD$, which implies that $F(D \mid S_3 = y)$ stochastically dominates $F(D \mid S_3 = \theta(y))$ in a first order sense. Therefore, by definition of first order stochastic dominance it follows that $E(D \mid S_3 = y) \geq E(D \mid S_3 = \theta(y))$.

B. Derivation of conditions that satisfy $\frac{\partial f_{21}(D \mid S_3 = \theta(y))}{\partial y} < \frac{\partial f_{21}(D \mid S_3 = y)}{\partial y}$

Using Leibniz's rule for differentiation, one can derive the following equations:

$$\frac{\partial E(D \mid S_3 = y)}{\partial y} = y f_{21}(y \mid S_3 = y) + \int_0^y D \frac{\partial f_{21}(D \mid S_3 = y)}{\partial y} dD \text{, where}$$

(B.1)

$$\frac{\partial f_{21}(D \mid S_3 = y)}{\partial y} = \frac{2g(y)g(y+D)}{G(y)^2} - \int_0^y \frac{4g(x)g(x+D)}{G(y)^3} dx$$

$$\frac{\partial E(D \mid S_3 = \theta(y))}{\partial y} = \theta(y) f_{21}(\theta(y) \mid S_3 = \theta(y)) \frac{\partial \theta(y)}{\partial y} + \int_0^{\theta(y)} D \frac{\partial f_{21}(D \mid S_3 = \theta(y))}{\partial y} dD \text{, where}$$

(B.2)

$$\frac{\partial f_{21}(D \mid S_3 = \theta(y))}{\partial y} = 2 \frac{\partial \theta}{\partial y} H(y, D) + \int_0^{\theta(y)} \frac{\partial \theta}{\partial y} J(y, D, x) dx$$

In equation B.2, J(.) and H(.) are functions that arise due to the differentiation of $f_{21}(.)$. Note that every term of equation B.2 is a multiple of $\frac{\partial\theta(y)}{\partial y}$. Therefore, as $\frac{\partial\theta(y)}{\partial y}$ tends to zero, so too will $\frac{\partial E(D \mid S_3 = \theta(y))}{\partial y}$. Furthermore, the dishonest bidder, and therefore the auctioneer, will find cheating most profitable if $\frac{\partial\theta(y)}{\partial y}$ is close to zero. For example, if the realization of the third bid were to increase, the collusive parties would lose money were they to increase the distance between the collusive bid and the low honest bid. Therefore, one would expect the marginal effect of the 3rd bid on the first spacing be smaller in instances of MNC than in honest auctions.

C. Expectation of the First Spacing, Conditioned on the Third Bid

1. Uniformly Distributed Bids

Recall the density of the first spacing, conditioned on the third bid. This density was shown to be

$$f_{21}(D \mid S_3 = y) = \frac{2}{G(y)^2}\int_0^y g(s)g(D+s)ds \tag{C.1}$$

First, suppose the bids are independent draws from the uniform distribution on [a,b]. In this scenario, $g(b) = \frac{1}{b-a}$, and $G(b) = \frac{x-a}{b-a}$ for values of b in the interval [a,b]. Substituting the density and distribution into equation A.1, we have the following:

$$f_{21}(D \mid S_3 = y) = 2\left(\frac{b-a}{y-a}\right)^2 \int_0^y \left(\frac{1}{b-a}\right)^2 ds \tag{C.2}$$

Solving the integral in equation C.2, we then find that the conditional density function reduces to

$$f_{21}(D \mid S_3 = y) = 2y\left(\frac{b-a}{y-a}\right)^2\left(\frac{1}{b-a}\right)^2 = \frac{2y}{(y-a)^2}$$

Applying the formula for the conditional expected value of a random variable, it is evident that the expected value of the first spacing conditioned on the third bid is given by

$$E_{21}(D \mid S_3 = y) = \int_0^y D f_{21}(D \mid S_3 = y)\,dD \tag{C.3}$$

Inserting the formula for the conditional density of the first spacing given above, we can then finish the algebra as follows:

$$\int_0^y D f_{21}(D \mid S_3 = y)\,dD = \int_0^y D\left(\frac{2y}{(y-a)^2}\right)dD = \frac{y^3}{(y-a)^2} \tag{C.4}$$

Thus, if the underlying bid function is uniform with a lower bound of 0, then the relationship between the first spacing and the third bid in a least squares regression equation will be linear. Interestingly enough, the conditional density does not depend on the upper bound, b. This makes perfect sense after considering what we have done in the above exercise—namely, to truncate the underlying density at a value less than the upper bound. Therefore, the upper bound of the distribution adds no new information to the conditional density.

2. Exponentially Distributed Bids

Next consider the scenario where bids are distributed exponentially, so that

$$g(b) = e^{-b}$$

$$G(b) = 1 - e^{-b}$$

Substituting these equations into C.1 above yields:

$$f_{21}(D \mid S_3 = y) = \frac{2}{(1-e^{-y})^2}\int_0^y e^{-s} e^{-s-D}\,ds \tag{C.5}$$

For notational convenience, let $\varphi = \dfrac{2}{(1-e^{-y})^2}$. Equation C.5 reduces as follows:

$$f_{21}(D \mid S_3 = y) = \varphi \int_0^y e^{-2s-D} ds$$

$$f_{21}(D \mid S_3 = y) = -\frac{1}{2}\varphi \int_0^y -2e^{-2s-D} ds$$

$$f_{21}(D \mid S_3 = y) = -\frac{1}{2}\varphi (e^{-2y-D} - e^{-D})$$

$$f_{21}(D \mid S_3 = y) = -\frac{1}{2}\varphi (e^{-2y} - 1)e^{-D}$$

$f_{21}(D \mid S_3 = y) = \dfrac{(1-e^{-2y})e^{-D}}{(1-e^{-y})^2}$, after substituting back for φ. We can now take advantage of the difference of squares in the numerator to complete the algebra and obtain equation C.6 below.

$$f_{21}(D \mid S_3 = y) = \frac{(1-e^{-y})e^{-D}}{1-e^{-y}} \tag{C.6}$$

After having derived equation C.6, we can substitute C.6 into equation C.3 above to obtain equation C.7. Solving C.7 will then yield the expectation of the first spacing given the third bid, when bids are distributed exponentially. These calculations are provided below, and result in equation C.8.

$$E_{21}(D \mid S_3 = y) = \int_0^y D \frac{(1+e^{-y})e^{-D}}{1-e^{-y}} dD \tag{C.7}$$

$$E_{21}(D \mid S_3 = y) = \left(\frac{1+e^{-y}}{1-e^{-y}}\right) \int_0^y De^{-D} dD$$

$$E_{21}(D \mid S_3 = y) = \left(\frac{1+e^{-y}}{1-e^{-y}}\right)(1 - ye^{-y} - e^{-y}) \qquad \text{(C.8)}$$

The last step in reaching equation C.8 is attained through integration by parts.

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