Exercise 16

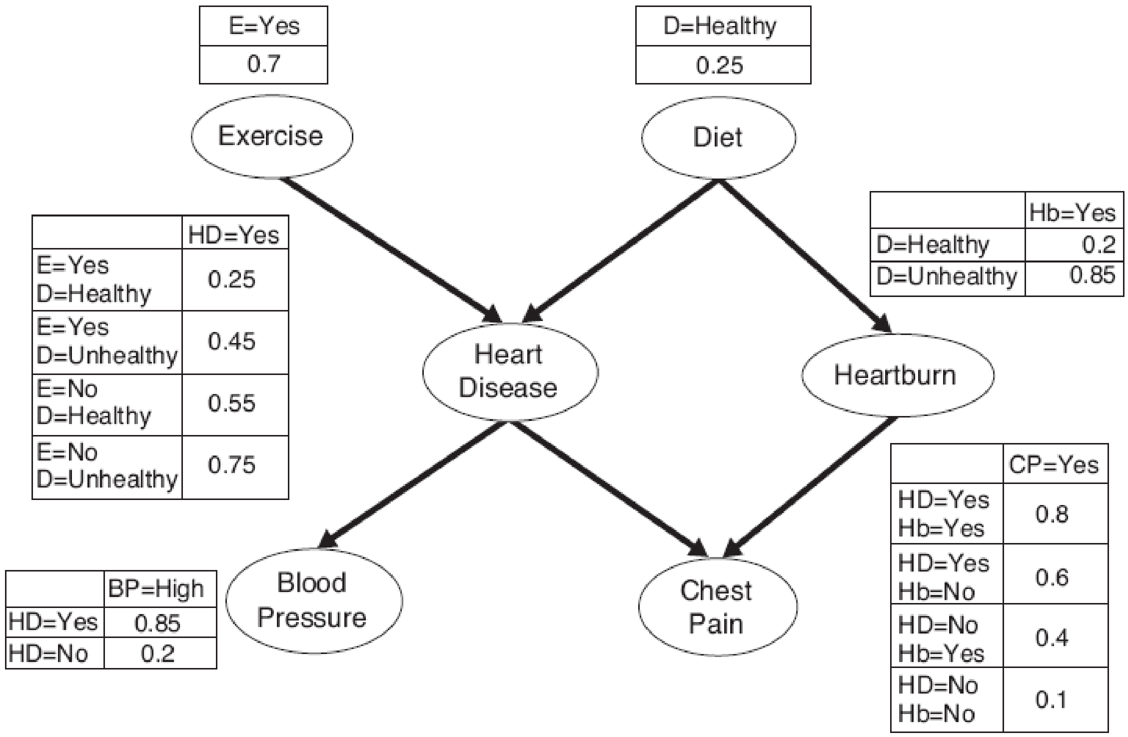

Figure 5.13. A Bayesian belief network for detecting heart disease and heartburn in patients.

Rules

P(A, B) = P(A) P(B), when A and B are independent
P(A, B) = P(A | B) P(B)
P(A | B) = P(B | A) P(A) / P(B)

(a)
P(HD = Yes)
= P(HD=Yes | E=Yes, D=Healthy) P(E=Yes) P(D=Healthy) +
P(HD=Yes | E=Yes, D=Unhealthy) P(E=Yes) P(D=Unhealthy) +
P(HD=Yes | E=No, D=Healthy) P(E=No) P(D=Healthy) +
P(HD=Yes | E=No, D=Unhealthy) P(E=No) P(D=Unhealthy)
= (0.25)(0.7)(0.25) + (0.45)(0.7)(0.75) + (0.55)(0.3)(0.25) + (0.75)(0.3)(0.75)
= 0.04375 + 0.23625 + 0.04125 + 0.16875
= ***0.49***

P(HD=Yes | D=Healthy)
= P(HD=Yes | D=Healthy, E=Yes) P(E=Yes) + P(HD=Yes | D=Healthy, E=No) P(E=No)
= 0.25 (0.7) + 0.55 (0.3)
= ***0.34***

(b)

Assumption of BBN: the probability of A is independent of non-descendants if the A's parents are known.

P(BP=High)
= P(BP=High | HD=Yes) P(HD=Yes) + P(BP=High | HD=No) P(HD=No)
= 0.85 (0.49) + 0.2 (0.51)
= 0.5185

P(Hb=Yes)
= P(Hb=Yes | D=Healthy) P(D=Healthy) + P(Hb=Yes | D=Unhealthy) P(D=Unhealthy)
= 0.2 (0.25) + 0.85 (0.75)
= 0.6875
P(Hb=No) = 1 – 0.6875 = 0.3125

P(CP=Yes | HD=Yes)
= P(CP=Yes | HD=Yes, Hb=Yes) P(Hb=Yes) + P(CP=Yes | HD=Yes, Hb=No) P(Hb=No)
= 0.8 (0.6875) + 0.6 (0.3125)
= 0.7375

P (CP=Y)
= P(CP=Y|HD=Y, Hb=Y)P(HD=Y, Hb=Y) + P(CP=Y|HD=Y, Hb=N)P(HD=Y, Hb=N) +
P(CP=Y|HD=N, Hb=Y)P(HD=N, Hb=Y) + P(CP=Y|HD=N, Hb=N)P(HD=N, Hb=N)
= 0.51

P (HD = Yes | BP = High, CP = Yes)
= P(HD=Yes, BP=High, CP=Yes) / P(BP=High, CP=Yes)
= P(BP=High | HD=Yes, ~~CP=Yes~~) P(HD=Yes, CP=Yes) / P(BP=High, CP=Yes)
= P(BP=High | HD=Yes) P(CP=Yes | HD=Yes) P(HD=Yes) / P(BP=High) P(CP=Yes)
= 0.85 (0.7375) (0.49) / 0.5185 (0.51)
= ***0.30***

CP is non-descendant of BP and BP's parent is known, therefore BP is independent of CP.