

Exercise 16

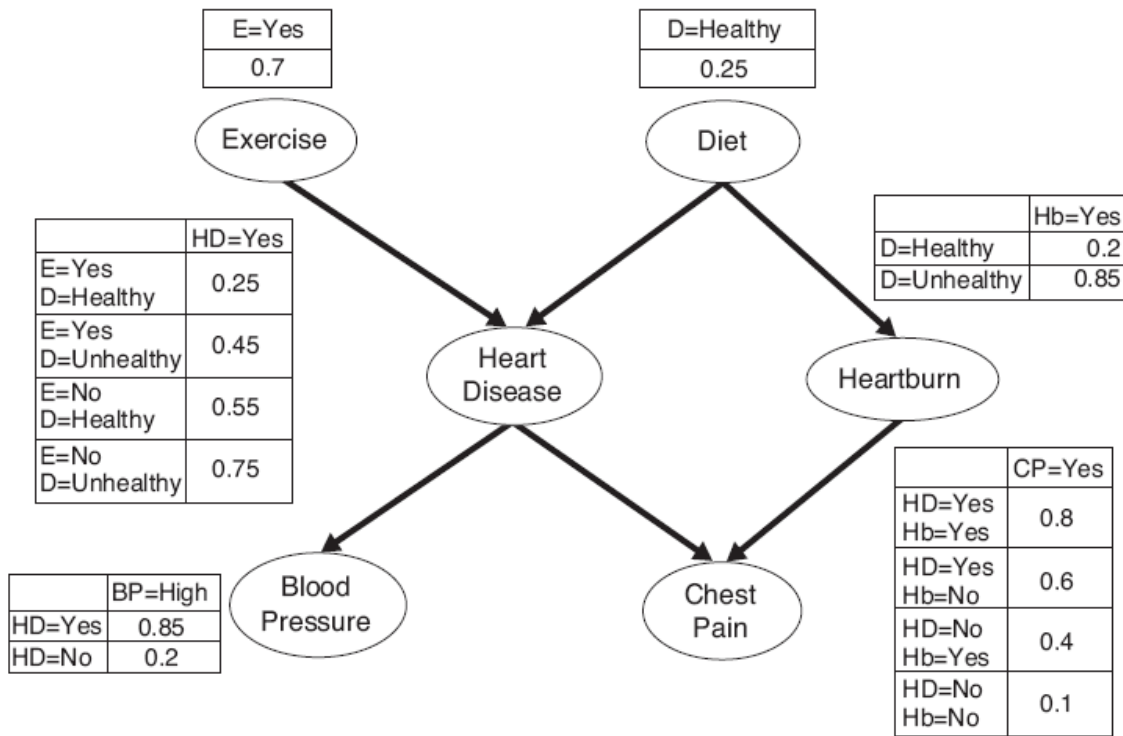


Figure 5.13. A Bayesian belief network for detecting heart disease and heartburn in patients.

Rules

$P(A, B) = P(A) P(B)$, when A and B are independent

$P(A, B) = P(A | B) P(B)$

$P(A | B) = P(B | A) P(A) / P(B)$

(a)

$P(\text{HD} = \text{Yes})$

$$\begin{aligned}
 &= P(\text{HD}=\text{Yes} | \text{E}=\text{Yes}, \text{D}=\text{Healthy}) P(\text{E}=\text{Yes}) P(\text{D}=\text{Healthy}) + \\
 &\quad P(\text{HD}=\text{Yes} | \text{E}=\text{Yes}, \text{D}=\text{Unhealthy}) P(\text{E}=\text{Yes}) P(\text{D}=\text{Unhealthy}) + \\
 &\quad P(\text{HD}=\text{Yes} | \text{E}=\text{No}, \text{D}=\text{Healthy}) P(\text{E}=\text{No}) P(\text{D}=\text{Healthy}) + \\
 &\quad P(\text{HD}=\text{Yes} | \text{E}=\text{No}, \text{D}=\text{Unhealthy}) P(\text{E}=\text{No}) P(\text{D}=\text{Unhealthy}) \\
 &= (0.25)(0.7)(0.25) + (0.45)(0.7)(0.75) + (0.55)(0.3)(0.25) + (0.75)(0.3)(0.75) \\
 &= 0.04375 + 0.23625 + 0.04125 + 0.16875 \\
 &= \mathbf{0.49}
 \end{aligned}$$

$P(\text{HD}=\text{Yes} | \text{D}=\text{Healthy})$

$$\begin{aligned}
 &= P(\text{HD}=\text{Yes} | \text{D}=\text{Healthy}, \text{E}=\text{Yes}) P(\text{D}=\text{Healthy}) P(\text{E}=\text{Yes}) + \\
 &\quad P(\text{HD}=\text{Yes} | \text{D}=\text{Healthy}, \text{E}=\text{No}) P(\text{D}=\text{Healthy}) P(\text{E}=\text{No}) \\
 &= 0.25 (0.25) (0.7) + 0.55 (0.25) (0.3) \\
 &= \mathbf{0.085}
 \end{aligned}$$

(b)

$$\begin{aligned} &P(\text{HD}=\text{Yes} \mid \text{BP}=\text{High}, \text{CP}=\text{Yes}) \\ &= P(\text{HD}=\text{Yes}, \text{BP}=\text{High}, \text{CP}=\text{Yes}) / P(\text{BP}=\text{High}, \text{CP}=\text{Yes}) \\ &= P(\text{HD}=\text{Yes}, \text{BP}=\text{High}) P(\text{CP}=\text{Yes}) / P(\text{BP}=\text{High}) P(\text{CP}=\text{Yes}) \\ &= P(\text{HD}=\text{Yes} \mid \text{BP}=\text{High}) \cancel{P(\text{BP}=\text{High})} P(\text{CP}=\text{Yes}) / \cancel{P(\text{BP}=\text{High})} P(\text{CP}=\text{Yes}) \\ &= 0.85 \end{aligned}$$