

## Exercise 14

There are 12 records used as a training set

First compute prior probabilities of each class

$$P(\text{mammal}) = 4/12$$

$$P(\text{reptile}) = 3/12$$

$$P(\text{fish}) = 2/12$$

$$P(\text{amphibian}) = 1/12$$

$$P(\text{bird}) = 2/12$$

### Test record X1: Porcupine

X1 = (Body Temperature = warm-blooded, Skin Cover = quills, Gives Birth = yes, Aquatic Creature = no, Aerial Creature = no, Has Legs = yes, Hibernates = yes)

$$\begin{aligned} P(X1 \mid \text{mammal}) &= P(\text{Body Temperature} = \text{warm-blooded} \mid \text{mammal}) \times P(\text{Skin Cover} = \text{quills} \mid \text{mammal}) \\ &\quad \times P(\text{Gives Birth} = \text{yes} \mid \text{mammal}) \times P(\text{Aquatic Creature} = \text{no} \mid \text{mammal}) \times P(\text{Aerial Creature} = \\ &\quad \text{no} \mid \text{mammal}) \times P(\text{Has Legs} = \text{yes} \mid \text{mammal}) \times P(\text{Hibernates} = \text{yes} \mid \text{mammal}) \\ &= 4/4 \times 0/4 = 0 \end{aligned}$$

(I stopped after computing the Skin Cover because 0 will make the entire computation 0 anyway)

$$\begin{aligned} P(X1 \mid \text{reptile}) &= P(\text{Body Temperature} = \text{warm-blooded} \mid \text{reptile}) \times P(\text{Skin Cover} = \text{quills} \mid \text{reptile}) \\ &\quad \times P(\text{Gives Birth} = \text{yes} \mid \text{reptile}) \times P(\text{Aquatic Creature} = \text{no} \mid \text{reptile}) \times P(\text{Aerial Creature} = \\ &\quad \text{no} \mid \text{reptile}) \times P(\text{Has Legs} = \text{yes} \mid \text{reptile}) \times P(\text{Hibernates} = \text{yes} \mid \text{reptile}) \\ &= 0/3 = 0 \end{aligned}$$

$$\begin{aligned} P(X1 \mid \text{fish}) &= P(\text{Body Temperature} = \text{warm-blooded} \mid \text{fish}) \times P(\text{Skin Cover} = \text{quills} \mid \text{fish}) \\ &\quad \times P(\text{Gives Birth} = \text{yes} \mid \text{fish}) \times P(\text{Aquatic Creature} = \text{no} \mid \text{fish}) \times P(\text{Aerial Creature} = \\ &\quad \text{no} \mid \text{fish}) \times P(\text{Has Legs} = \text{yes} \mid \text{fish}) \times P(\text{Hibernates} = \text{yes} \mid \text{fish}) \\ &= 0/2 = 0 \end{aligned}$$

$$\begin{aligned} P(X1 \mid \text{amphibian}) &= P(\text{Body Temperature} = \text{warm-blooded} \mid \text{amphibian}) \times P(\text{Skin Cover} = \text{quills} \mid \text{amphibian}) \\ &\quad \times P(\text{Gives Birth} = \text{yes} \mid \text{amphibian}) \times P(\text{Aquatic Creature} = \text{no} \mid \text{amphibian}) \times P(\text{Aerial} \\ &\quad \text{Creature} = \text{no} \mid \text{amphibian}) \times P(\text{Has Legs} = \text{yes} \mid \text{amphibian}) \times P(\text{Hibernates} = \text{yes} \mid \text{amphibian}) \\ &= 0/1 = 0 \end{aligned}$$

$$\begin{aligned} P(X1 \mid \text{bird}) &= P(\text{Body Temperature} = \text{warm-blooded} \mid \text{bird}) \times P(\text{Skin Cover} = \text{quills} \mid \text{bird}) \\ &\quad \times P(\text{Gives Birth} = \text{yes} \mid \text{bird}) \times P(\text{Aquatic Creature} = \text{no} \mid \text{bird}) \times P(\text{Aerial Creature} = \\ &\quad \text{no} \mid \text{bird}) \times P(\text{Has Legs} = \text{yes} \mid \text{bird}) \times P(\text{Hibernates} = \text{yes} \mid \text{bird}) \\ &= 2/2 \times 0/2 = 0 \end{aligned}$$

The posterior probability for all the classes is 0 because their class-conditional probability is 0.

So Naïve Bayesian Classification is not able to classify the record.

## Test record X2: Eel

$X_2 = (\text{Body Temperature} = \text{cold-blooded}, \text{Skin Cover} = \text{scales}, \text{Gives Birth} = \text{no}, \text{Aquatic Creature} = \text{yes}, \text{Aerial Creature} = \text{no}, \text{Has Legs} = \text{no}, \text{Hibernates} = \text{no})$

$$\begin{aligned} P(X_2 | \text{mammal}) &= P(\text{Body Temperature} = \text{cold-blooded} | \text{mammal}) \times P(\text{Skin Cover} = \text{scales} | \text{mammal}) \\ &\quad \times P(\text{Gives Birth} = \text{no} | \text{mammal}) \times P(\text{Aquatic Creature} = \text{yes} | \text{mammal}) \times P(\text{Aerial Creature} = \\ &\quad \text{no} | \text{mammal}) \times P(\text{Has Legs} = \text{no} | \text{mammal}) \times P(\text{Hibernates} = \text{no} | \text{mammal}) \\ &= 0/4 \end{aligned}$$

$$\begin{aligned} P(X_2 | \text{reptile}) &= P(\text{Body Temperature} = \text{cold-blooded} | \text{reptile}) \times P(\text{Skin Cover} = \text{scales} | \text{reptile}) \\ &\quad \times P(\text{Gives Birth} = \text{no} | \text{reptile}) \times P(\text{Aquatic Creature} = \text{yes} | \text{reptile}) \times P(\text{Aerial Creature} = \\ &\quad \text{no} | \text{reptile}) \times P(\text{Has Legs} = \text{no} | \text{reptile}) \times P(\text{Hibernates} = \text{no} | \text{reptile}) \\ &= 2/3 \times 2/3 \times 2/3 \times 0/3 = 0 \end{aligned}$$

$$\begin{aligned} P(X_2 | \text{fish}) &= P(\text{Body Temperature} = \text{cold-blooded} | \text{fish}) \times P(\text{Skin Cover} = \text{scales} | \text{fish}) \\ &\quad \times P(\text{Gives Birth} = \text{no} | \text{fish}) \times P(\text{Aquatic Creature} = \text{yes} | \text{fish}) \times P(\text{Aerial Creature} = \\ &\quad \text{no} | \text{fish}) \times P(\text{Has Legs} = \text{no} | \text{fish}) \times P(\text{Hibernates} = \text{no} | \text{fish}) \\ &= 2/2 \times 2/2 \times 1/2 \times 2/2 \times 2/2 \times 2/2 \times 2/2 = 1/2 \end{aligned}$$

$$\begin{aligned} P(X_2 | \text{amphibian}) &= P(\text{Body Temperature} = \text{cold-blooded} | \text{amphibian}) \times P(\text{Skin Cover} = \text{scales} | \text{amphibian}) \\ &\quad \times P(\text{Gives Birth} = \text{no} | \text{amphibian}) \times P(\text{Aquatic Creature} = \text{yes} | \text{amphibian}) \times P(\text{Aerial} \\ &\quad \text{Creature} = \text{no} | \text{amphibian}) \times P(\text{Has Legs} = \text{no} | \text{amphibian}) \times P(\text{Hibernates} = \text{no} | \text{amphibian}) \\ &= 1/1 \times 0/1 = 0 \end{aligned}$$

$$\begin{aligned} P(X_2 | \text{bird}) &= P(\text{Body Temperature} = \text{cold-blooded} | \text{bird}) \times P(\text{Skin Cover} = \text{scales} | \text{bird}) \\ &\quad \times P(\text{Gives Birth} = \text{no} | \text{bird}) \times P(\text{Aquatic Creature} = \text{yes} | \text{bird}) \times P(\text{Aerial Creature} = \\ &\quad \text{no} | \text{bird}) \times P(\text{Has Legs} = \text{no} | \text{bird}) \times P(\text{Hibernates} = \text{no} | \text{bird}) \\ &= 0/2 \end{aligned}$$

The posterior probability for class fish is  $P(\text{fish} | X_2) = (a) \times (2/12) \times (.5) = .083a$  where  $a = 1/P(X_2)$  is a constant term. The posterior probability for all the other classes is 0 because their class-conditional probability is 0.

Since  $P(\text{fish} | X_2)$  is greater than all the others, the record is classified as fish.

### Test record X3: Salamander

X3 = (Body Temperature = cold-blooded, Skin Cover = none, Gives Birth = no, Aquatic Creature = semi, Aerial Creature = no, Has Legs = yes, Hibernates = yes)

$$\begin{aligned}P(X3 | \text{mammal}) &= P(\text{Body Temperature} = \text{cold-blooded} | \text{mammal}) \times P(\text{Skin Cover} = \text{none} | \text{mammal}) \\ &\quad \times P(\text{Gives Birth} = \text{no} | \text{mammal}) \times P(\text{Aquatic Creature} = \text{semi} | \text{mammal}) \times P(\text{Aerial Creature} \\ &\quad = \text{no} | \text{mammal}) \times P(\text{Has Legs} = \text{yes} | \text{mammal}) \times P(\text{Hibernates} = \text{yes} | \text{mammal}) \\ &= 0/4 = 0\end{aligned}$$

$$\begin{aligned}P(X3 | \text{reptile}) &= P(\text{Body Temperature} = \text{cold-blooded} | \text{reptile}) \times P(\text{Skin Cover} = \text{none} | \text{reptile}) \\ &\quad \times P(\text{Gives Birth} = \text{no} | \text{reptile}) \times P(\text{Aquatic Creature} = \text{semi} | \text{reptile}) \times P(\text{Aerial Creature} = \\ &\quad = \text{no} | \text{reptile}) \times P(\text{Has Legs} = \text{yes} | \text{reptile}) \times P(\text{Hibernates} = \text{yes} | \text{reptile}) \\ &= 3/3 \times 0/3 = 0\end{aligned}$$

$$\begin{aligned}P(X3 | \text{fish}) &= P(\text{Body Temperature} = \text{cold-blooded} | \text{fish}) \times P(\text{Skin Cover} = \text{none} | \text{fish}) \\ &\quad \times P(\text{Gives Birth} = \text{no} | \text{fish}) \times P(\text{Aquatic Creature} = \text{semi} | \text{fish}) \times P(\text{Aerial Creature} = \\ &\quad = \text{no} | \text{fish}) \times P(\text{Has Legs} = \text{yes} | \text{fish}) \times P(\text{Hibernates} = \text{yes} | \text{fish}) \\ &= 2/2 \times 0/2 = 0\end{aligned}$$

$$\begin{aligned}P(X3 | \text{amphibian}) &= P(\text{Body Temperature} = \text{cold-blooded} | \text{amphibian}) \times P(\text{Skin Cover} = \text{none} | \text{amphibian}) \\ &\quad \times P(\text{Gives Birth} = \text{no} | \text{amphibian}) \times P(\text{Aquatic Creature} = \text{semi} | \text{amphibian}) \times P(\text{Aerial} \\ &\quad \text{Creature} = \text{no} | \text{amphibian}) \times P(\text{Has Legs} = \text{yes} | \text{amphibian}) \times P(\text{Hibernates} = \text{yes} | \text{amphibian}) \\ &= 1/1 \times 1/1 \times 1/1 \times 1/1 \times 1/1 \times 1/1 \times 1/1 = 1\end{aligned}$$

$$\begin{aligned}P(X3 | \text{bird}) &= P(\text{Body Temperature} = \text{cold-blooded} | \text{bird}) \times P(\text{Skin Cover} = \text{none} | \text{bird}) \\ &\quad \times P(\text{Gives Birth} = \text{no} | \text{bird}) \times P(\text{Aquatic Creature} = \text{semi} | \text{bird}) \times P(\text{Aerial Creature} = \\ &\quad = \text{no} | \text{bird}) \times P(\text{Has Legs} = \text{yes} | \text{bird}) \times P(\text{Hibernates} = \text{yes} | \text{bird}) \\ &= 0/2 = 0\end{aligned}$$

The posterior probability for class amphibian is  $P(\text{amphibian} | X3) = (a) \times (1/12) \times (1) = .083a$  where  $a = 1/P(X3)$  is a constant term. The posterior probability for all the other classes is 0 because their class-conditional probability is 0.

Since  $P(\text{amphibian} | X3)$  is greater than all the others, the record is classified as amphibian.